LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2013**

MT 6603/MT 6600 - COMPLEX ANALYSIS

Date : 25/04/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

PART – A

Answer ALL questions: (10 x2=20)

1. Show that the function $f(z) = \bar{z}$ is nowhere differentiable.
2. Show that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic.
3. State Cauchy-Goursat's theorem.
4. State Morera's theorem
5. Write the Taylor's series expansion of f(z)=cosz.
6. Define removable and essential singularities.
7. Define residue of a function at a point.
8. State Rouche's theorem.
9. Define conformal mapping.
10. Define a bilinear transformation.

PART – B

Answer any FIVE questions: (5x8=40)

11. Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin , although Cauchy-Riemann equations are satisfied at the origin.
12. Find the regular function whose imaginary part is $e^{-x}(x\cos y + y\sin y)$
13. Find the radius of convergence of the power series $f(z) = \sum_{0}^{\infty} \frac{z^n}{2^n + 1}$.
14. State and prove Liouville's theorem.
15. Expand $f(z) = \frac{z}{(z-1)(z-3)}$ as laurent's series valid in the following regions(i) $1 < |z| < 3$ (ii) $0 < |z-1| < 2$.
16. Classify the singularity of the function $f(z) = \frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$.

17. Apply cauchy residue theorem to show that $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz = 4\pi i$

where C is the positively oriented circle $|z| = 3$,

18. Find the bilinear transformation which maps the points z = -2,0,2 into the points w=0,i,-i respectively.

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PART - C

Answer any TWO questions: **(2x20=40)**

19. a) State and prove the sufficient conditions for f(z) to be differentiable at a point.

b) If f(z) is an analytic function show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (12 + 8)

20. State and prove Cauchy's integral formula and use it to evaluate $\int_C \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$. (10+10)

21. a) State and prove Laurent's theorem.

b) Using contour integration along the unit circle, evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\sin\theta}$. (10+10)

22. a) Prove that any bilinear transformation which maps the unit circle $|z| = 1$ onto the unit circle $|w| = 1$ can be written in the form $w = e^{i\lambda}\left(\frac{z - \alpha}{\overline{\alpha} z - 1}\right)$ where λ is real number.

b) State and prove Cauchy's residue theorem. (12+8)

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